

Exercise 14

Differentiate.

$$y = \frac{\sin t}{1 + \tan t}$$

SolutionUse the quotient rule to differentiate y .

$$\begin{aligned} y' &= \frac{dy}{dt} \\ &= \frac{d}{dt} \left(\frac{\sin t}{1 + \tan t} \right) \\ &= \frac{\left[\frac{d}{dt}(\sin t) \right] (1 + \tan t) - \left[\frac{d}{dt}(1 + \tan t) \right] (\sin t)}{(1 + \tan t)^2} \\ &= \frac{(\cos t)(1 + \tan t) - (\sec^2 t)(\sin t)}{(1 + \tan t)^2} \\ &= \frac{\cos t + \cos t \tan t - (\tan^2 t + 1)(\sin t)}{(1 + \tan t)^2} \\ &= \frac{\cos t + \cancel{\sin t} - \tan^2 t \sin t - \cancel{\sin t}}{(1 + \tan t)^2} \\ &= \frac{\cos t - \tan^2 t \sin t}{(1 + \tan t)^2} \end{aligned}$$